

考試科目	21614, 41114, 41214, 微積分 41414, 41514 系所別 41614, 41814	商院共同科、經濟學系	考試時間	7月11日(三) 第四節
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2. (20 pts) Find the following integrals.

(a) (5 pts) $\int_0^{\pi} \sin(x) dx.$

(b) (5 pts) $\int_0^1 (2x+3)^{1/2} dx.$

(c) (5 pts) $\int_0^1 x \ln(x) dx.$

(d) (5 pts) $\int_D (2x+3y)d(x,y)$, where $D = \{(x,y) : |x| + |y| \leq 1\}.$

命中&相似題目：微積分學習要訣 P.4-29 例4(1)

第四章 不定積分之求法 4-29

$$\begin{aligned} \therefore \text{原式} &= \frac{1}{2}x^2 \sin^{-1}x - \int \frac{x^2}{2\sqrt{1-x^2}} dx \stackrel{x=\sin\theta \text{ (詳§4-6)}}{=} \frac{1}{2}x^2 \sin^{-1}x - \frac{1}{2} \int \frac{\sin^2\theta}{\cos\theta} \cos\theta d\theta \\ &= \frac{1}{2}x^2 \sin^{-1}x - \frac{1}{4} \left(\theta - \frac{1}{2} \sin 2\theta \right) + c \\ &= \frac{1}{2}x^2 \sin^{-1}x - \frac{1}{4} \sin^{-1}x + \frac{1}{4}x\sqrt{1-x^2} + c. \end{aligned}$$

說例 3 基本題 求 $\int \ln x dx = ?$ (經典必考)

[解]

<法一> 令 $u = \ln x, dv = dx$

$$du = \frac{1}{x} dx, v = x$$

$$\therefore \text{原式} = x \ln x - \int 1 dx = x \ln x - x + c.$$

<法二> 令 $t = \ln x$, 則 $e^t = x, dt = \frac{1}{x} dx \rightarrow e^t dt = dx, \therefore \text{原式} = \int te^t dt$

再令 $u = t, dv = e^t dt$

$$du = dt, v = e^t$$

$$\text{故 } \int te^t dt = te^t - \int e^t dt = te^t - e^t + c = x(\ln x - 1) + c.$$

類 求 $\int \ln(x+a) dx = ?$ $a > 0$ 為常數 (經典常考)

答：令 $u = \ln(x+a), dv = dx$

$$du = \frac{1}{x+a} dx, v = x$$

$$\begin{aligned} \therefore \text{原式} &= x \ln(x+a) - \int \frac{x}{x+a} dx = x \ln(x+a) - \int \left(1 - \frac{a}{x+a} \right) dx \\ &= x \ln(x+a) - x + a \ln|x+a| + c. \end{aligned}$$

說例 4 基本題 求(1) $\int x \ln x dx = ?$ (2) $\int \frac{\ln x}{x^3} dx = ?$