

國立臺北大學 107 學年度日間學士班暨進修學士班轉學生招生考試試題

系 別：統計學系日間學士班 2、3 年級

科 目：微積分

3. Let  $f(x) = \frac{1}{1+x^2}$ ,  $0 \leq x \leq 1$  and  $P = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$  be a partition of  $[0, 1]$ .
- (a) (7%) Calculate the upper sum  $U(f, p)$  and the lower sum  $L(f, p)$  of  $f$  over the partition  $P$ . Rounded the answers to the third decimal place.
- (b) (6%) Approximate  $\int_0^1 \frac{1}{1+x^2} dx$  by the Simpson's rule with  $n=4$  and round to the third decimal place.
- (c) (7%) Evaluate  $\lim_{n \rightarrow \infty} \left[ \sum_{i=1}^n \frac{n}{n^2 + i^2} \right]$ .

命中&相似題目：微積分學習要訣 P.5-50 第 4 題

5-50 微積分學習要訣

$$\begin{aligned}
 \text{<法二>上下同乘 } \frac{1}{n} \text{ 得 } \lim_{n \rightarrow \infty} \frac{\sum_{i=1}^{2n} \left(\frac{1}{2}\right)^p \frac{1}{n}}{\sum_{i=1}^n \left(\frac{1}{2} + \frac{i}{2n}\right)^p \frac{1}{n}} &= \frac{\int_0^2 \left(\frac{1}{2}x\right)^p dx}{\int_0^1 \left(\frac{1}{2} + \frac{1}{2}x\right)^p dx} \\
 &= \frac{\left(\frac{1}{2}\right)^p \left[ \frac{1}{1+p} x^{p+1} \right]_0^2}{\left[ \frac{2}{1+p} \left(\frac{1}{2} + \frac{1}{2}x\right)^{p+1} \right]_0^1} = \frac{\left(\frac{1}{2}\right)^p \cdot \frac{1}{1+p} 2^{p+1}}{\frac{2}{1+p} - \frac{2}{1+p} \left(\frac{1}{2}\right)^{p+1}} \\
 &= \frac{1}{1 - \left(\frac{1}{2}\right)^{p+1}}.
 \end{aligned}$$

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※精選習作※

- $\lim_{n \rightarrow \infty} \frac{1}{n^2} \left( \sqrt{1 + \frac{1}{n}} + 2\sqrt{1 + \frac{2}{n}} + \dots + n\sqrt{1 + \frac{n}{n}} \right) = ?$
- $\lim_{n \rightarrow \infty} \left[ \frac{(2n)!}{n!n^n} \right]^{\frac{1}{n}} = ?$
- $\lim_{n \rightarrow \infty} \frac{1}{n^{p+1}} (1^p + 2^p + \dots + n^p) = ? \quad p > 0$  (常考題)
- $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{n}{n^2 + k^2} = ?$
- $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \sin \frac{kt}{n} = ?$
- $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\sqrt{n^2 + k^2}} = ?$
- $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} \left( \frac{1}{\sqrt{n+1}} + \frac{1}{\sqrt{n+2}} + \dots + \frac{1}{\sqrt{n+n}} \right) = ?$
- $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{\sqrt{n^2 - k^2}}{n^2} = ?$
- $\lim_{n \rightarrow \infty} \left( \frac{1}{\sqrt{n^2 + 2n}} + \frac{1}{\sqrt{n^2 + 4n}} + \dots + \frac{1}{\sqrt{n^2 + 2n^2}} \right) = ?$

5. Evaluate the following integrals.

(a) (7%)  $\int_1^{\infty} \frac{\ln x}{x^2} dx$

命中&相似題目：微積分學習要訣 P.4-30 類(2)

4-30 微積分學習要訣

[解]

(1) 令  $u = \ln x$  ,  $dv = x dx$

$$du = \frac{1}{x} dx \quad , \quad v = \frac{1}{2} x^2$$

$$\therefore \text{原式} = \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x dx = \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + c \circ$$

(2) 令  $u = \ln x$  ,  $dv = \frac{dx}{x^3}$

$$du = \frac{1}{x} dx \quad , \quad v = -\frac{1}{2x^2}$$

$$\therefore \text{原式} = -\frac{1}{2x^2} \ln x + \int \frac{1}{2x^3} dx = -\frac{1}{2x^2} \ln x - \frac{1}{4x^2} + c \circ$$

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**類** 求  $\int x^3 \ln x dx = ?$     $\int \frac{\ln x}{x^2} dx = ?$

答：令  $u = \ln x$  ,  $dv = x^3 dx$

$$du = \frac{1}{x} dx \quad , \quad v = \frac{1}{4} x^4$$

$$\therefore \text{原式} = \frac{1}{4} x^4 \ln x - \int \frac{1}{4} x^3 dx = \frac{1}{4} x^4 \ln x - \frac{1}{16} x^4 + c$$

同理，令  $u = \ln x$  ,  $dv = \frac{dx}{x^2}$

$$du = \frac{1}{x} dx \quad , \quad v = -\frac{1}{x}$$

$$\therefore \text{原式} = -\frac{\ln x}{x} + \int \frac{1}{x^2} dx = -\frac{\ln x}{x} - \frac{1}{x} + c \circ$$

但有些題目須經“二次以上”的分部積分步驟才可完成！現將這些題目整理成如下之**速解法**說明之。

說例 5 基本題	求 $\int x^2 e^x dx = ?$
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[解] 此類問題有速解法如下：