

※ 注意：請於試卷上「非選擇題作答區」標明大題及小題題號，並依序作答。

Any device with computer algebra system is prohibited during the exam.

PART 1 : Fill in the blanks.

- Only answers will be graded.
- Each answer must be clearly labeled on the answer sheet.
- 4 points are assigned to each blank.

1. (a) $\lim_{x \rightarrow \infty} (1 + \frac{3}{x})^{[x]} = \underline{(1)}$, where $[x]$ is the greatest integer which is less than or equal to x .

(b) $\lim_{x \rightarrow 0} (\cos ax)^{\frac{1}{1-\cos bx}} = \underline{(2)}$, where a, b are constants and $ab \neq 0$.

(c) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{2n\sqrt{1 - (\frac{i}{2n})^2}} = \underline{(3)}$.

命中&相似題目：講義 p.6-22 Ex23; p.6-33 Exercise 33 (相似度 90%)

Ex 23.

Evaluate $\lim_{n \rightarrow \infty} \sin \frac{\pi}{n} \cdot \sum_{k=1}^n \frac{1}{2 + \cos \frac{k\pi}{n}}$.

Solution. $\frac{\pi}{\sqrt{3}}$

Exercise 23.

Compute $\lim_{n \rightarrow \infty} \sum_{k=1}^n \sqrt{\frac{4k}{n}} \cdot \frac{4}{n}$

Ans.

[觀點 1]: 令 $f(x) = \sqrt{x}$, $x_k = \frac{4k}{n}$, $k = 1, 2, \dots, n$, $\Delta x = \frac{4}{n}$. 則

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \sqrt{\frac{4k}{n}} \cdot \frac{4}{n} = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \cdot \Delta x = \int_0^4 \sqrt{x} dx = \frac{2}{3} x^{3/2} \Big|_0^4 = \frac{16}{3}. \quad \#$$

[觀點 2]: 令 $f(x) = \sqrt{4x}$, $x_k = \frac{k}{n}$, $k = 1, 2, \dots, n$, $\Delta x = \frac{1}{n}$. 則

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \sqrt{\frac{4k}{n}} \cdot \frac{4}{n} = 4 \lim_{n \rightarrow \infty} \sum_{k=1}^n \sqrt{\frac{k}{n}} \cdot \frac{1}{n} = 4 \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \cdot \Delta x = 4 \int_0^1 \sqrt{x} dx = 4 \cdot \frac{2}{3} x^{3/2} \Big|_0^1 = \frac{16}{3}. \quad \#$$

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6. (a) $\iint_D y \, dA = \underline{\quad (15) \quad}$, where D is the region in the first quadrant that lies between the circles $x^2 + y^2 = 4$ and $(x-1)^2 + y^2 = 1$.

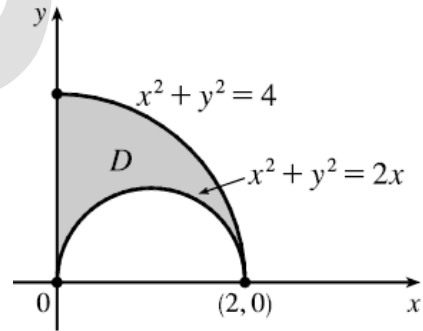
命中&相似題目：講義 p.12-20 Exercise3 (相似度 99%)

Exercise 3.

Evaluate $\iint_D x \, dA$, where D is the region in the first quadrant that lies between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 2x$

Ans.

$$\begin{aligned} \iint_D x \, dA &= \iint_{\substack{x^2+y^2 \leq 4 \\ x, y \geq 0}} x \, dA - \iint_{\substack{(x-1)^2+y^2 \leq 1 \\ y \geq 0}} x \, dA \\ &= \int_0^{\pi/2} \int_0^2 r^2 \cos \theta \, dr \, d\theta - \int_0^{\pi} \int_0^1 r^2 \cos \theta \, dr \, d\theta \\ &= \int_0^{\pi/2} \frac{21}{3} (8 \cos \theta) \, d\theta - \int_0^{\pi} \frac{1}{3} (8 \cos^3 \theta) \, d\theta \\ &= \frac{8}{3} - \frac{\pi}{2}. \quad \# \end{aligned}$$



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5. Let the region R be enclosed by the curves $y = x^2$ and $y = 2 - x^2$. Find the volume of the solid obtained by rotating the region R about $x = 1$.

命中&相似題目：講義 p.7-20 Homework7.2 #5 (相似度 100%)

Homework 7.2

1-6, Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

1. $y = x + 1, y = 0, x = 0, x = 2$, about the x -axis.
2. $x = y - y^2, x = 0$, about the y -axis
3. $y = x^2, x = y^2$, about $y = 1$.
4. $y = \frac{1}{x}, y = 0, x = 1, x = 2$, about y -axis.

5. $y = x^2, y = 2 - x^2$, about $x = 1$.

7. Evaluate $\iint_R \cos\left(\frac{y-x}{y+x}\right) dA$ where R is the trapezoidal region with vertices $(1, 0), (2, 0), (0, 2)$ and $(0, 1)$.

命中&相似題目：講義 p.12-22 Ex16. (相似度 100%)

Ex 16.

Evaluate $\iint_R \cos\left(\frac{y-x}{y+x}\right) dA$, where R is the trapezoidal region with vertices $(1, 0), (2, 0), (0, 2)$ and $(0, 1)$.

Solution. $\frac{3}{2} \sin 1$

8. Let $f(x, y) = x^4 + y^4 - 4xy + 1$. Find local maxima, local minima, and saddle points of $f(x, y)$.

命中&相似題目：講義 p.11-35 Exercise9. (相似度 99%)

Exercise 9.

Find all the local maxima, local minima, and saddle points of the function $f(x, y) = 4xy - x^4 - y^4$.

Ans.

$$\begin{cases} f_x = 4y - 4x^3 = 0 \\ f_y = 4x - 4y^3 = 0 \end{cases} \Rightarrow \begin{cases} y = x^3 \\ x = y^3 \end{cases} \Rightarrow x = x^9 \Rightarrow x(x^8 - 1) = 0 \Rightarrow x = 0, \pm 1.$$

$\therefore (x, y) = (0, 0), (1, 1), (-1, -1)$ 為臨界點.

$$f_{xx} = -12x^2, f_{yy} = -12y^2, f_{xy} = 4, \Delta(x, y) = f_{xx}f_{yy} - (f_{xy})^2 = 144x^2y^2 - 4$$

$\therefore \Delta(0, 0) = -4 < 0$ $\therefore (0, 0)$ 為鞍點.

$\therefore \Delta(1, 1) = 140 > 0$ 且 $f_{xx}(1, 1) = -12 < 0$ $\therefore f(1, 1) = 4 - 1 - 1$ 為局部最大值.

$\therefore \Delta(-1, -1) = 140 > 0$ 且 $f_{xx}(-1, -1) = -12 < 0$ $\therefore f(-1, -1) = 4 - 1 - 1$ 為局部最大值. #

9. Find the absolute maximum value and absolute minimum value of $f(x, y) = x^4 + y^4 - 4xy + 1$ on the disk $x^2 + y^2 \leq 1$.

命中&相似題目：講義 p.11-28 Ex22. (相似度 90%)

Ex 22.

Find the extreme values of the function $f(x, y) = x^2 + 2y^2$ on the region $x^2 + y^2 \leq 1$.

Solution. Max.=2, min.=0

10. Solve the differential equation $xy' = y + x^2 \sin x$ with $y(\pi) = 2\pi$.

命中&相似題目：講義 p.9-5 Ex5. (相似度 90%)

Ex 5.

Find the solution of the initial-value problem $x^2y' + xy = 1, x > 0, y(1) = 2$.

Solution. $y(x) = \frac{\ln x + 2}{x}$