

7. How many local extreme values does the function  $f(x, y) = 10xye^{-(x^2+y^2)}$  have?  
 Answer : \_\_\_\_\_

命中&相似題目：講義 p.11-33 Homewor11 #3 (相似度 80%)

Homework 11

1. Let  $f(x, y, z) = 1 + \frac{x^2}{6} + \frac{y^2}{12} + \frac{z^2}{18}$ , and  $\mathbf{n} = \frac{1}{\sqrt{3}}(1, 1, 1)$ , find  $D_{\mathbf{u}}f(1, 2, 3)$ .

2. Find the gradient of the function  $f(x, y) = \arctan \frac{x}{y}$  at point  $(0, 1)$ .

3. Find all the local maxima, local minima, and saddle points of  $f(x, y) = xe^{-\frac{x^2+y^2}{2}}$ .

2.

a. Determine whether the series  $\sum_{n=1}^{\infty} (-1)^n \ln\left(1 + \frac{1}{n}\right)$  diverges or converges conditionally or converges absolutely and give reasons for your answer. (6 points)

b. Show that if  $\sum_{n=1}^{\infty} a_n$  converges, then  $\sum_{n=1}^{\infty} \left(\frac{3 + \sin(a_n)}{5}\right)^n$  converges. (6 points)

命中&相似題目：講義 p.10-37 Exercise2 (a) (相似度 80%)

Exercise 2.

Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

(a)  $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$       (b)  $\sum_{n=1}^{\infty} \left(\frac{1-n}{2+3n}\right)^n$       (c)  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (n!)^2}{(2n)!}$

Ans.

(a) (1) 令  $a_n = \frac{1}{\ln n}$ .  $\therefore a_n = \frac{1}{\ln n} > \frac{1}{\ln(n+1)} = a_{n+1}$ ,  $\forall n = 1, 2, \dots$ .  $\therefore a_n$  為遞減

此外，顯然的， $\lim_{n \rightarrow \infty} \frac{1}{\ln n} = 0$

$\therefore \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$  收斂. #

(2) 考慮  $\sum_{n=2}^{\infty} \left| \frac{(-1)^n}{\ln n} \right| = \sum_{n=2}^{\infty} \frac{1}{\ln n}$  .

$\because \frac{1}{\ln n} > \frac{1}{n}$  且  $\sum_{n=1}^{\infty} \frac{1}{n}$  發散  $\therefore$  由比較法知， $\sum_{n=2}^{\infty} \left| \frac{(-1)^n}{\ln n} \right|$  發散. #

綜合 (1) 與 (2) 知， $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$  為條件收斂.

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