

臺灣綜合大學系統 109 學年度學士班轉學生聯合招生考試試題

科目名稱	微積分 A	類組代碼	共同考科
		科目碼	E0011
※本項考試依簡章規定所有考科均「不可」使用計算機。		本科試題共計 2 頁	
答題時，請詳述計算過程，否則將不予計分。			
1. (10 pts) Evaluate the following limits if they exist.			
(a) $\lim_{n \rightarrow \infty} \frac{3n+2}{2n+1}$			
(b) $\lim_{x \rightarrow 0} \frac{\cos(2x)-1}{x^2}$			

命中&相似題目：講義 p.1-19 Ex17(a) & Remark (4). (相似度 95%)

<p>Ex 17. Evaluate the limit if it exists.</p> <p>(a) $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x}$ (b) $\lim_{x \rightarrow 0} \frac{\sin 7x}{4x}$ (c) $\lim_{x \rightarrow \infty} \frac{3x^2 + 5}{5x + 3} \sin \frac{2}{x}$ (d) $\lim_{x \rightarrow 0} \frac{\tan 6x}{\sin 2x}$</p>
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Solution. (a) 0 (b) $\frac{7}{4}$ (c) $\frac{6}{5}$ (d) 3

Remark.

以下極限式可當作常識記起來，當遇到填充題或較複雜的極限計算時可直接使用：

(1) $\lim_{x \rightarrow 0} \frac{\sin ax}{bx} = \lim_{x \rightarrow 0} \frac{ax}{\sin bx} = \lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx} = \frac{a}{b}$; (2) $\lim_{x \rightarrow 0} \frac{\tan ax}{bx} = \lim_{x \rightarrow 0} \frac{ax}{\tan bx} = \frac{a}{b}$;

(3) $\lim_{x \rightarrow 0} \frac{\tan ax}{\sin bx} = \lim_{x \rightarrow 0} \frac{\sin ax}{\tan bx} = \lim_{x \rightarrow 0} \frac{\tan ax}{\tan bx} = \frac{a}{b}$; (4) $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$, $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2} = -\frac{1}{2}$.

7. (10 pts) Let S be the surface defined by the equation

$$x \cos(xy) + z^2 y^4 - 7xz = 1$$

and $P(0,1,1)$ be a point on S . Find an equation that defines the tangent plane to S at P and

命中&相似題目：講義 p.11-18 Ex15 & p.11-24 Exercise 3 (相似度 80%)

Ex 15.

Find the equations of the tangent plane and normal line at the point $(-2,1,-3)$ to the ellipsoid

$$\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3.$$

Solution. $3x - 6y + 2z = -18$, $\frac{x+2}{-1} = \frac{y-1}{2} = \frac{z+3}{-2/3}$

Exercise 3.

曲面 $x^2 + \cos(xy) + yz + x = 0$ 在 $(0, 1, -1)$ 處的切平面方程為

(A) $x - y + z = -2$ (B) $x + y + z = 0$ (C) $x - 2y + z = -3$ (D) $x - y - z = 0$

Ans.

令 $f(x, y, z) = x^2 + \cos(xy) + yz + x \Rightarrow \nabla f(0, 1, -1) = (1, -1, 1)$

\therefore 切平面方程為 $x - y + z = -2$ 選 (A). #

8. (10 pts) Evaluate the double integral

$$\iint_R (y - x) dA$$

where $R = \{(x, y) \in \mathbb{R}^2 : 1 \leq x^2 + y^2 \leq 4, x \geq 0\}$.

命中&相似題目：講義 p.12-12 Ex9 & p.12-27 Homework 12 (相似度 80%)

Ex 9.

Evaluate $\iint_R (3x + 4y^2) dA$, where R is the region in the upper half-plane bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

Solution. $\frac{15}{2}\pi$

20. Evaluate $\iint_R \arctan(y/x) dA$, where $R = \{(x, y) | 1 \leq x^2 + y^2 \leq 4, 0 \leq y \leq x\}$.

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科目名稱	微積分 B	類組代碼	共同考科
		科目碼	E0012

1. (10 points) Find the following limits.

(a)

$$\lim_{x \rightarrow 2} \frac{x + 4}{x - 7}$$

(b)

$$\lim_{x \rightarrow 0} (1 - 7x)^{\frac{4}{x}}$$

命中&相似題目：講義 p.1-23 Ex21(a) (相似度 90%)

Ex 21.

Evaluate

(a) $\lim_{x \rightarrow 0} (1-2x)^{1/x}$ (b) $\lim_{x \rightarrow \infty} \left[\frac{x^2}{(x-a)(x+b)} \right]^x$ (c) $\lim_{x \rightarrow 0} (\cos(2x))^{1/x^2}$ <107. 政大應數轉>

Solution. (a) e^{-2} (b) e^{a-b} (c) e^{-2}

8. (10 points) Find $(f^{-1})'(5)$ for

$$f(x) = x^5 + 2x^3 + 2x.$$

命中&相似題目：講義 p.3-47 Ex23 (相似度 90%)

Ex 23.

(a) If $f(x) = 3x^3 + 4x^2 + 6x + 5$, find $(f^{-1})'(5)$.

(b) If $f(x) = 3 + x^2 + \tan(\pi x/2)$, $-1 < x < 1$, find $(f^{-1})'(3)$.

Solution. (a) $\frac{1}{7}$ (b) $\frac{7}{6}$

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科目名稱	微積分 C	類組代碼	共同考科
		科目碼	E0013

1. (10 points) Find the following limits:

(a) (5 points) $\lim_{x \rightarrow 1} \frac{e^x - 1}{x}$

(b) (5 points) $\lim_{x \rightarrow -\infty} (\sqrt{x^2 + x + 1} + x)$

命中&相似題目：講義 p.1-15 Ex130 (相似度 100%)

Ex 13.

Evaluate (a) $\lim_{x \rightarrow \infty} (\sqrt{9x^2 + x} - 3x)$ (b) $\lim_{x \rightarrow -\infty} (\sqrt{x^2 + x + 1} + x)$

Solution. (a) $\frac{1}{6}$ (b) $\frac{-1}{2}$

6. (10 points) Consider the power series $\sum_{n=1}^{\infty} \frac{(-3)^n(x+2)^n}{\sqrt{n+1}}$. Find its maximal domain of convergence.

命中&相似題目：講義 p.10-42 Ex19 (相似度 99%)

Ex 19.

Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$.

Solution. $(-1/3, 1/3]$

8. (10 points) Find the absolute maximum of $f(x, y) = e^{-xy}$ on the region $x^2 + 4y^2 \leq 1$.

命中&相似題目：講義 p.11-34 Exercise7 (相似度 95%)

Exercise 7.

Find the extreme values of f on the region $f(x, y) = e^{-xy}$, $2x^2 + y^2 \leq 4$.

Ans.

$$\begin{cases} f_x = -ye^{-xy} = 0 \\ f_y = -xe^{-xy} = 0 \end{cases} \Rightarrow (x, y) = (0, 0) \text{ 為臨界點且在限制區域 } 2x^2 + y^2 \leq 4 \text{ 內.}$$

考慮 $F(x, y, \lambda) = e^{-xy} + \lambda(2x^2 + y^2 - 4)$, 並令

$$\begin{cases} F_x = -ye^{-xy} + 4\lambda x = 0 \\ F_y = -xe^{-xy} + 2\lambda y = 0 \\ F_\lambda = 2x^2 + y^2 - 4 = 0 \end{cases} \Rightarrow \begin{cases} \lambda = \frac{ye^{-xy}}{4x} \\ \lambda = \frac{xe^{-xy}}{2y} \\ 2x^2 + y^2 - 4 = 0 \dots\dots (*) \end{cases}$$

則 $\frac{ye^{-xy}}{4x} = \frac{xe^{-xy}}{2y} \Rightarrow 4x^2 = 2y^2 \Rightarrow 2x^2 = y^2$ 代入 (*) 得

$$2x^2 + 2x^2 - 4 = 0 \Rightarrow x = \pm 1, y = \pm 1.$$

$$f(0,0) = 1, f(1,1) = f(-1,-1) = e^{-1}, f(-1,1) = f(1,-1) = e.$$

\therefore 最大值為 e , 最小值為 e^{-1} . #