7. How many local extreme values does the function $f(x,y)=10xye^{-(x^2+y^2)}$ have?
Answer :________

命中&相似題目 ： 講義 p.11-33 Homewor11 #3 (相似度 80%)

Homework 11

1. Let $f(x,y,z)=1+\dfrac{x^2}{6}+\dfrac{y^2}{12}+\dfrac{z^2}{18}$, and $\mathbf{n}=\dfrac{1}{\sqrt{3}}(1,1,1)$, find $D_{\mathbf{u}}f(1,2,3)$.

2. Find the gradient of the function $f(x,y)=\arctan\dfrac{x}{y}$ at point $(0,1)$.

3. Find all the local maxima, local minima, and saddle points of $f(x,y)=xe^{-\frac{x^2+y^2}{2}}$.

2.

a. Determine whether the series $\sum_{n=1}^{\infty}(-1)^n\ln\left(1+\frac{1}{n}\right)$ diverges or converges conditionally or converges absolutely and give reasons for your answer. (6 points)

b. Show that if $\sum_{n=1}^{\infty}a_n$ converges, then $\sum_{n=1}^{\infty}\left(\frac{3+\sin(a_n)}{5}\right)^n$ converges. (6 points)

命中&相似題目 ： 講義 p.10-37 Exercise2 (a) (相似度 80%)

Exercise 2.

Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

(a) $\sum_{n=2}^{\infty}\dfrac{(-1)^n}{\ln n}$ (b) $\sum_{n=1}^{\infty}\left(\dfrac{1-n}{2+3n}\right)^n$ (c) $\sum_{n=1}^{\infty}\dfrac{(-1)^{n+1}(n!)^2}{(2n)!}$

Ans.

(a) (1) 令 $a_n=\dfrac{1}{\ln n}$. $\because a_n=\dfrac{1}{\ln n}>\dfrac{1}{\ln(n+1)}=a_{n+1}$, $\forall n=1,2,\cdots$. $\therefore a_n$ 為遞減

此外，顯然的， $\lim\limits_{n\to\infty}\dfrac{1}{\ln n}=0$

$\therefore \sum_{n=2}^{\infty}\dfrac{(-1)^n}{\ln n}$ 收斂. #

(2) 考慮 $\sum_{n=2}^{\infty}\left|\frac{(-1)^n}{\ln n}\right|=\sum_{n=2}^{\infty}\frac{1}{\ln n}$.

$\because \frac{1}{\ln n}>\frac{1}{n}$ 且 $\sum_{n=1}^{\infty}\frac{1}{n}$ 發散 $\therefore$ 由比較法知， $\sum_{n=2}^{\infty}\left|\frac{(-1)^n}{\ln n}\right|$ 發散. #

綜合 (1)與 (2) 知， $\sum_{n=2}^{\infty}\frac{(-1)^n}{\ln n}$ 為條件收斂.

注意：考試開始鈴響前，不可以翻閱試題

台灣聯合大學系統 107 學年度學士班轉學考試題

考試科目：微積分

參考用

組別：A2

1. Determine the limits of integration where $a \le b$ such that $\int_a^b (x^2 - 16)\,dx$ has minimal value. Answer :________

命中&相似題目：微積分學習要訣 P.6-9 例 7 之類題

類 求二曲線 $\begin{cases} x+y=0 \\ y^2+3y=x \end{cases}$ 所包圍區域的面積？

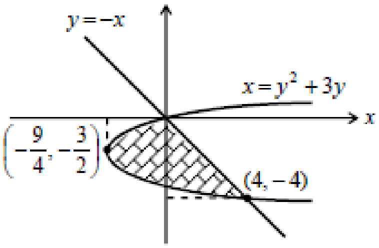
答：求出 $\begin{cases} x+y=0 \\ y^2+3y=x \end{cases}$ 之交點

為 $(4,-4)$, $(0,0)$

並繪圖如右：

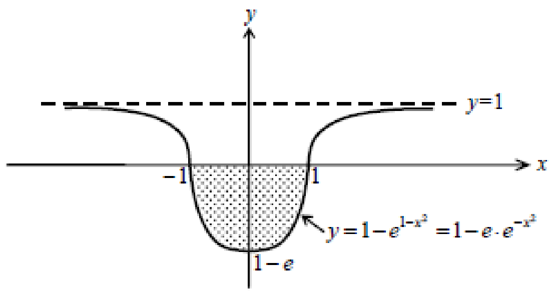
$A=\int_{-4}^{0}\left[-y-(y^2+3y)\right]dy$

$=\frac{32}{3}$。



說例 7 台聯大	有二數 a、b，$a<b$ 使 $\int_a^b (1-e^{1-x^2})dx$ 有最小值，求 $b-a=$?

[解]此類題目是掛羊頭賣狗肉！其實是考繪圖，將 $y=1-e^{1-x^2}$ 繪圖如下：



僅在區間 $[-1,1]$ 內其面積為負，$\therefore a=-1,\ b=1$，故 $b-a=2$。

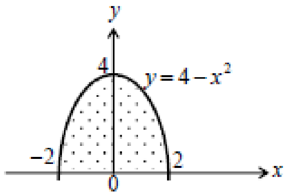
* * *

類 有二數 a、b，且 $a<b$，使 $\int_a^b (4-x^2)dx$ 有最大值，求 $b-a=$?

答：繪 $y=4-x^2$ 之圖形得知

欲得最大面積，由圖形知

$a=-2,\ b=2$，$\therefore b-a=4$。



2. Evaluate $\int_0^{\pi/2} \sqrt{1-\sin x}\,dx$.

命中&相似題目：微積分學習要訣 P.4-19 例 20

$$\text{原式} = \int \sqrt{1-u}\cdot\frac{du}{\sqrt{1-u^2}} = \int \frac{du}{\sqrt{1+u}} = 2\sqrt{1+u}+c = 2\sqrt{1+\cos\theta}+c\text{。}$$

*　　*　　*

類 求 $\int \sqrt{1+\cos\theta}\,d\theta = ?$

答：

<法一>原式 $= \int \sqrt{2\cos^2\frac{\theta}{2}}\,d\theta = \sqrt{2}\int \cos\frac{\theta}{2}\,d\theta = 2\sqrt{2}\sin\frac{\theta}{2}+c$。

<法二> $u=\cos\theta$，則 $du=\sin\theta\,d\theta \to d\theta = \frac{du}{\sin\theta} = \frac{du}{\sqrt{1-\cos^2\theta}} = \frac{du}{\sqrt{1-u^2}}$

$$\text{原式} = \int \sqrt{1+u}\cdot\frac{du}{\sqrt{1-u^2}} = \int \frac{du}{\sqrt{1-u}} = -2\sqrt{1-u}+c = -2\sqrt{1-\cos\theta}+c\text{。}$$

說例 20 技巧題	求 $\int \sqrt{1-\sin\theta}\,d\theta = ?$

[解]

<法一>原式 $= \int \sqrt{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}-2\cos\frac{\theta}{2}\sin\frac{\theta}{2}}\,d\theta = \int \sqrt{(\sin\frac{\theta}{2}-\cos\frac{\theta}{2})^2}\,d\theta$

$$= \int (\sin\frac{\theta}{2}-\cos\frac{\theta}{2})\,d\theta = 2(-\cos\frac{\theta}{2}-\sin\frac{\theta}{2})+c\text{。}$$

或表為 $\int \sqrt{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}-2\cos\frac{\theta}{2}\sin\frac{\theta}{2}}\,d\theta = \int \sqrt{(\cos\frac{\theta}{2}-\sin\frac{\theta}{2})^2}\,d\theta$

$$= \int (\cos\frac{\theta}{2}-\sin\frac{\theta}{2})\,d\theta = 2(\sin\frac{\theta}{2}+\cos\frac{\theta}{2})+c\text{。}$$

<法二> $u=\sin\theta$，則 $du=\cos\theta\,d\theta \to d\theta = \frac{du}{\cos\theta} = \frac{du}{\sqrt{1-\sin^2\theta}} = \frac{du}{\sqrt{1-u^2}}$

$$\text{原式} = \int \sqrt{1-u}\cdot\frac{du}{\sqrt{1-u^2}} = \int \frac{du}{\sqrt{1+u}} = 2\sqrt{1+u}+c = 2\sqrt{1+\sin\theta}+c\text{。}$$

*　　*　　*

類 求 $\int \sqrt{1+\sin\theta}\,d\theta = ?$

答：