

題號：19  
科目：微積分(B)

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共2頁之第1頁

9.  $\int_0^1 \frac{1}{(x^2 + 1)^2} dx = \underline{\hspace{2cm}} \text{ (10) }.$

命中&相似題目：微積分學習要訣 P.4-55

第四章 不定積分之求法 4-55

$$= \frac{1}{4a} \left[ -\frac{1}{2} \ln(x^2 + 2ax + 2a^2) + \tan^{-1} \left( \frac{x+a}{a} \right) \right] + \frac{1}{4a} \left[ \frac{1}{2} \ln(x^2 - 2ax + 2a^2) + \tan^{-1} \left( \frac{x-a}{a} \right) \right] + C.$$

<b>說例 9</b> <b>漂亮題</b>	求 $\int \frac{1}{(x^2 + 1)^2} dx = ?$
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[解] 令  $x = \tan \theta, dx = \sec^2 \theta d\theta$

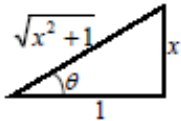
原式 =  $\int \frac{\sec^2 \theta}{(\tan^2 \theta + 1)^2} d\theta = \int \frac{\sec^2 \theta}{\sec^4 \theta} d\theta$

$= \int \cos^2 \theta d\theta = \int \frac{1 + \cos 2\theta}{2} d\theta$

$= \frac{\theta}{2} + \frac{1}{4} \sin 2\theta + C$

$= \frac{1}{2} \tan^{-1} x + \frac{1}{4} \cdot 2 \cdot \frac{x}{\sqrt{x^2 + 1}} \cdot \frac{1}{\sqrt{x^2 + 1}} + C$

$= \frac{1}{2} \tan^{-1} x + \frac{x}{2(x^2 + 1)} + C.$



\* \* \*

<b>類</b>	求 $\int \frac{1}{(x^2 - 1)^2} dx = ?$
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答：令  $x = \sec \theta, dx = \sec \theta \tan \theta d\theta$

原式 =  $\int \frac{\sec \theta \tan \theta}{(\tan^2 \theta)^2} d\theta = \int \cot^3 \theta \sec \theta d\theta$

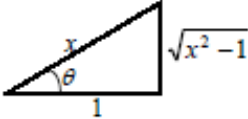
$= \int \cot^2 \theta \csc \theta d\theta = \int (\csc^2 \theta - 1) \csc \theta d\theta$

$= \int \csc^3 \theta d\theta - \int \csc \theta d\theta$

$= -\frac{1}{2} [\csc \theta \cot \theta + \ln |\csc \theta + \cot \theta|] + \ln |\csc \theta + \cot \theta| + C$

$= -\frac{1}{2} \csc \theta \cot \theta + \frac{1}{2} \ln |\csc \theta + \cot \theta| + C$

$= -\frac{1}{2} \frac{x}{x^2 - 1} + \frac{1}{2} \ln \left| \frac{x+1}{x-1} \right| + C.$



13. If  $a$  and  $b$  are positive constants and if  $\max\{p, q\}$  denotes the maximum between the numbers  $p$  and  $q$ , the iterated integral  $\int_0^a \int_0^b e^{\max\{b^2x^2, a^2y^2\}} dy dx = \underline{\hspace{2cm}} (17)$ .

命中&相似題目：微積分考前總攻略 P.9-77

### 微積分考前總攻略

9-77

$$\begin{aligned}
 &= \int_0^1 \int_0^1 \frac{1}{yz} \left[ yz - \frac{1}{2}y^2z^2 + \frac{1}{3}y^3z^3 - \dots \right] dy dz \\
 &= \int_0^1 \int_0^1 \left[ 1 - \frac{1}{2}yz + \frac{1}{3}y^2z^2 - \dots \right] dy dz \\
 &= \int_0^1 \left[ y - \frac{1}{4}y^2z + \frac{1}{9}y^3z^2 - \dots \right]_{y=0}^{y=1} dz \\
 &= \int_0^1 \left[ 1 - \frac{1}{4}z + \frac{1}{9}z^2 - \dots \right] dz = \left[ z - \frac{1}{8}z^2 + \frac{1}{27}z^3 - \dots \right]_0^1 \\
 &= 1 - \frac{1}{8} + \frac{1}{27} - \dots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}.
 \end{aligned}$$

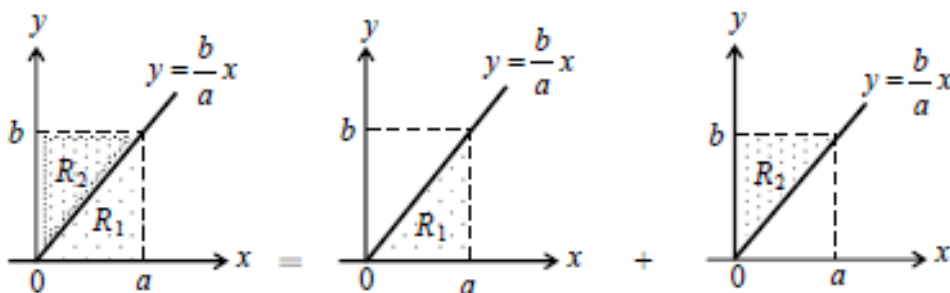
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### 試題【中央研】

求  $\int_0^b \int_0^a e^{\max\{b^2x^2, a^2y^2\}} dx dy = ?$

答

將積分區域分割如下：



$$\begin{aligned}
 \text{原式} &= \int_0^a \int_0^{\frac{b}{a}x} e^{b^2x^2} dy dx + \int_0^{\frac{a}{b}y} \int_0^b e^{a^2y^2} dx dy \\
 &= \int_0^a \left[ ye^{b^2x^2} \right]_{y=0}^{y=\frac{b}{a}x} dx + \int_0^{\frac{a}{b}y} \left[ xe^{a^2y^2} \right]_{x=0}^{x=b} dy = \int_0^a \frac{b}{a} xe^{b^2x^2} dx + \int_0^{\frac{a}{b}y} \frac{b}{a} ye^{a^2y^2} dy \\
 &= \left[ \frac{1}{2ab} e^{b^2x^2} \right]_0^a + \left[ \frac{1}{2ab} e^{a^2y^2} \right]_0^{\frac{a}{b}y} = \frac{1}{2ab} (e^{a^2b^2} - 1) + \frac{1}{2ab} (e^{a^2b^2} - 1)
 \end{aligned}$$